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UNIVERSITÉ DE VERSAILLES ST-QUENTIN-EN-YVELINES
université PARIS-SACLAY

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Working Paper CEMOTEV n° 06-2017

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Université de Versailles Saint-Quentin en Yvelines
47 Boulevard Vauban
78047 Guyancourt Cedex
www.cemotev.uvsq.fr
Tel : 01 39 25 57 00 / Mail : cemotev@uvsq.fr

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Fabrice Barthélémy†, Mathieu Martin‡and Ashley Piggins§

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Abstract

Donald Trump won the 2016 U.S. Presidential election with fewer popular votes than Hillary Clinton. This is the fourth time this has happened, the others being 1876, 1888 and 2000. In our earlier paper "The architecture of the Electoral College, the House size effect, and the referendum paradox" (Electoral Studies 34 (2014) 111-118), we analyzed these earlier elections (and others) and showed how the electoral winner can often depend on the size of the House of Representatives. A sufficiently larger House would have given electoral victories to the winner of the popular vote in both 1876 and 2000. An exception is the election of 1888. In this note we show that Trump's victory in 2016 is like Harrison's in 1888, and unlike 1876 and 2000. This note updates the analysis of our earlier paper to include the 2016 election.

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†Department of Economics, CEMOTEV, Université de Versailles Saint-Quentin-en-Yvelines, Guyancourt, France. Email: fabrice.barthelemy@uvsq.fr

‡THEMA, Université de Cergy-Pontoise, 33 boulevard du Port, 95011 Cergy Pontoise Cedex, France. Email: mathieu.martin@u-cergy.fr

§J.E. Cairnes School of Business and Economics and the Whitaker Institute, National University of Ireland Galway, University Road, Galway, Ireland. Email: ashley.piggins@nuigalway.ie

1 Introduction

For the fourth time in American history the winner of the popular vote in the presidential election is not the same as the winner of the electoral vote. Donald J. Trump obtained 46.09% of the popular vote (62,984,825 votes) to Hillary Clinton's 48.18% (65,853,516 votes).[1] Despite this, Clinton only obtained 227 votes in the Electoral College to Trump's 304 with seven "faithless" electors. The outcome was particularly ironic since Trump himself declared on Twitter in 2012, "The electoral college is a disaster for a democracy".[2] As in 1876, 1888 and 2000 the winner of the electoral vote was a Republican. Following Nurmi (1998), we refer to these outcomes as examples of a "referendum paradox", i.e. a situation where the electoral winner is not the same as the winner under a direct popular vote (a referendum).[3]

In Barthélémy, Martin and Piggins (2014) we analyzed these presidential elections (and others) and noted how the electoral winner can occasionally depend on particular (and somewhat arbitrary) features of the Electoral College; features that we referred to as "architecture". More specifically, we showed in these elections (using a device called a representation graph) how variations in the size of the House of Representatives, *inter alia*, can change the electoral winner without anyone changing how they vote. This can occur because the number of electors each state has depends, in part, on the

[1] Data for the 2016 election comes from "Official 2016 Presidential Election Results", Federal Election Commission, January 30, 2017. Available at http://www.fec.gov/pubrec/fe2016/2016presgeresults.pdf. Date retrieved April 20, 2017. Other election data used in this note comes from David Leap's comprehensive *Atlas of U.S. Presidential Elections* available at http://uselectionatlas.org.

[2] Tweet by Donald J. Trump (@realDonaldTrump) on 6th November 2012 at 8:45 pm. Tweet available at https://goo.gl/Kza0Ii. Date retrieved May 3, 2017.

[3] Miller (2012) uses the term "electoral inversion" rather than "referendum paradox", and we understand that this is the more common term in U.S. political science. Note that the 1824 presidential election could also be viewed as paradoxical, in that the popular vote winner, Andrew Jackson, lost the election to John Quincy Adams. Jackson won a plurality of electoral votes, but not a majority, due to the presence of two additional candidates (William Crawford and Henry Clay), both of whom carried some states. On foot of the Twelfth Amendment, the House of Representatives subsequently voted Adams president.

number of seats it has in the House of Representatives. Increasing the size of the House, other things equal, will increase the total number of seats to be apportioned to the states. Since the method of apportionment used in the Unites States is "House-monotone",[4] no state will end up with fewer seats as House size increases. This additional representation will directly affect the number of electors in each state and so, potentially, the outcome of the election.

This "House size effect" was first noted for the 2000 election by Neubauer and Zeitlin (2003) and a theoretical explanation of it was given by Miller (2014). Neubauer and Zeitlin show that in the 2000 election, a sufficiently large House would have given electoral victory to Al Gore instead of George W. Bush.[5] Barthélémy, Martin and Piggins (2014) showed that a House size effect was also present in the 1876 election. In that election, a sufficiently large House would have given electoral victory to Samuel Tilden instead of Rutherford Hayes. This election is notable not only in that Tilden obtained more popular votes than Hayes, but also in that he won an absolute majority (51%) of the popular vote. We can treat 1876 and 2000 as similar in that sufficiently larger House sizes would have resulted in the popular vote winner being elected.

An exception to this is the election of 1888. Grover Cleveland obtained more popular votes than Benjamin Harrison and lost the electoral vote. Barthélémy, Martin and Piggins (2014) show that no House size effect is present in that election, so changing the size of the House would not have been to Cleveland's advantage. They called the referendum paradox of 1888 "entrenched" in that none of the architectural variations they consider would

[4]Balinski and Young (2001) is the definitive treatment of the apportionment problem.

[5]Neubauer and Zeitlin show that the relationship between House size and electoral winner is not necessarily monotonic. Note that the size of the House was fixed at 435 in 1911. There was a temporary increase to 437 at the time of admission of Alaska and Hawaii as states in 1959. However, for the apportionment of seats on foot of the 1960 census, which took effect for the election in 1962, the number of seats reverted to 435.

have resulted in Cleveland's election.[6]

The purpose of this note is to update Barthélémy, Martin and Piggins (2014) with the results of the 2016 presidential election which was also paradoxical. We show that Trump's victory was "entrenched" like Harrison's in 1888, and unlike Bush in 2000 and Hayes in 1876.[7] No variations that we consider would result in Clinton's election, in particular there is no House size effect.

2 Representation graph

Our basic analytical device is a representation graph (Barthélémy, Martin and Piggins (2014)). This is a simple two-dimensional graph, with the number of electors measured on the horizontal axis, and the proportion of electoral votes for the Democratic candidate measured on the vertical axis. Currently the number of electors is 538, one for each of the 100 members of the Senate and 435 members of the House of Representatives, plus three for the District of Columbia.

We use election data to graph how the proportion of electoral votes for the Democrat changes as the size of the House increases or decreases. For example, as House size increases (other things equal) the number of electors will increase, and we move rightward along the horizontal axis of Figure 1. For each additional seat in the House, we compute a new congressional apportionment and calculate the proportion of electoral votes for the Democrat

[6] Barthélémy, Martin and Piggins vary not only the size of the House in their election simulations, but also the method of apportionment, the number of electors a state has by virtue of its senators (currently this is two), and the lower bound (or floor) on the number of electors a state has by virtue of its representatives in the House (currently this is one). The Constitution determines these latter two values, whereas Congress determines House size and the method of apportionment. The method used to allocate a state's electoral votes is determined by state legislatures. In this note, we do not consider varying the method of apportioning seats to states as it has no bearing on our observation about the 2016 election.

[7] Note that in all of these paradoxical elections, the outcomes favored Republicans.

on foot of this.[8] This allows us to graph the effect of House size on electoral outcomes.

Our simulations are performed using the software GAUSS.[9] If the plotted graph ever crosses 50% then there is a House size effect. The Electoral College, given that particular House size, will elect a different president. Our graphs are plotted for fixed intervals of five additional electors, although the software computes an apportionment for increments of one.

To aid the exposition, we first reproduce Figure 1 of Barthélémy, Martin and Piggins (2014). This figure refers to the 2000 presidential election.[10]

A vertical line is drawn at 538 electors and a 50% dashed horizontal line is also plotted; k refers to the number of electoral votes each state has by virtue of its senators and m is the lower bound on the number of electors a state has by virtue of its representatives. As noted above, currently $k = 2$ and $m = 1$. This corresponds to the bold black line in Figure 1. Four representation graphs are plotted, one for each of the possible parameter values listed in the legend.

The referendum paradox of 2000 can be identified from the figure; the bold black line crosses the vertical line beneath the dashed 50% line. Gore loses the electoral vote. Second, a sufficiently large House would have ensured victory for Gore. The representation graph lies above 50% as House size increases. The graph also exhibits the non-monotonicity identified by Neubauer and Zeitlin; the winner oscillates back and forth for a certain interval of House sizes.[11] However, as House size increases even further the

[8] In our simulations we treat the District of Columbia as receiving the same number of electoral votes as the least populous state.

[9] http://www.aptech.com

[10] Note that in this election there was no "split" of electoral votes in Maine and Nebraska. Maine and Nebraska select one elector within each congressional district by popular vote, and select their remaining two electors by the statewide plurality winner. All other states operate a "winner-takes-all" rule under which the candidate with the largest popular vote in the state takes all of the state's electoral votes.

[11] Neubauer and Zeitlin show that if the size of the House is less than 491, then Bush is always the winner, and if it is greater than 597 then Gore is always the winner (with,

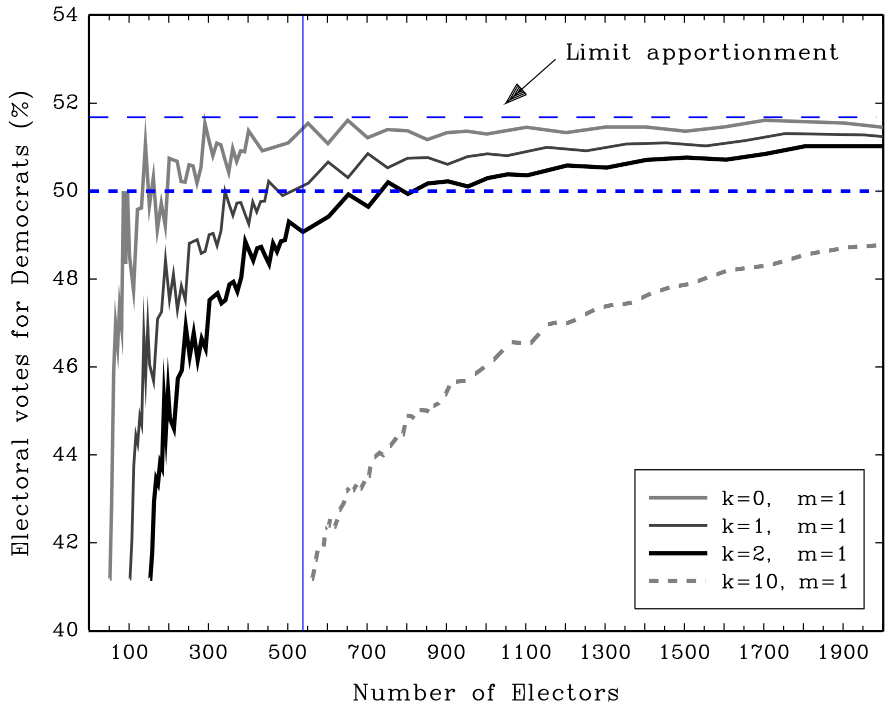

Figure 1: 2000 election.

graph approaches a second horizontal, dashed-line called "limit apportionment". This horizontal line is drawn at the electoral vote percentage that the Democratic candidate would obtain if the size of the House was equal to the total apportionment population.[12] In this hypothetical situation, the size of the House is so large that each state, in effect, casts a number of electoral votes equal to its population.[13] As a matter of logic, then, under the "winner-takes-all" rule, the limit apportionment electoral vote percentage must equal the percentage of the U.S. apportionment population that resides in states carried by the Democratic candidate. Indeed, in the 2000 election, Gore carried states that accounted for 51.68% of the total apportionment population.[14] The corresponding representation graphs for the 1876 election look similar to Figure 2.[15] The electoral vote loser and popular vote winner (Samuel Tilden) would have won the 1876 election with a sufficiently large House size.

Figure 2 is the corresponding figure for the 2016 presidential election.[16] Importantly, we can see in Figure 2 that there is no House size effect. The four representation graphs plotted in Figure 2 are safely below 50% on the vertical axis. Unlike 1876 and 2000, a larger House would not have been advantageous to the popular vote winner. Moreover, the limit apportionment is approximately 43.7%.[17]

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somewhat surprisingly, a tie at 655). Between these two numbers, sometimes Bush wins, sometimes Gore wins, and sometimes there is a tie.

[12] The apportionment population includes the resident population for the 50 states, as ascertained by the decennial census, plus counts of overseas U.S. military and federal civilian employees. The apportionment population excludes the population of the District of Columbia.

[13] The two additional votes cast by each state by virtue of their Senators become insignificant when the total vote cast is so large.

[14] Miller (2014).

[15] See Figure 5(a) in Barthélémy, Martin and Piggins (2014).

[16] In 2016, Maine split its electoral votes, three to Clinton and one to Trump. In our simulations, we treat all of these electoral votes as cast for Clinton. There was no split in Nebraska. We also treat the "faithless" electors as voting as pledged.

[17] Census data used for the 2010 apportionment is available at https://www.census.gov/data/tables/2010/dec/2010-apportionment-data.html.

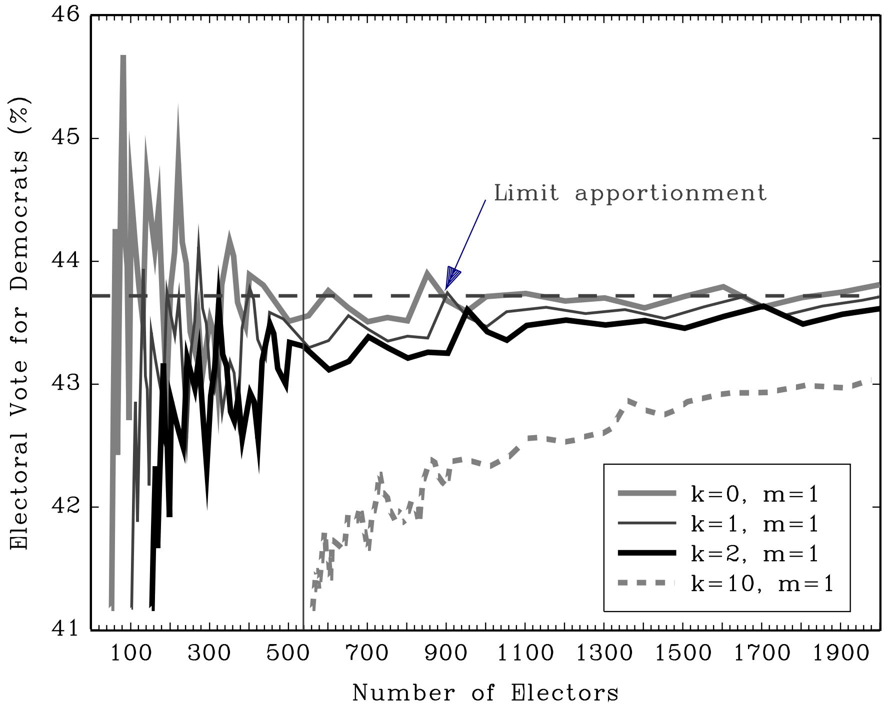

Figure 2: 2016 election.

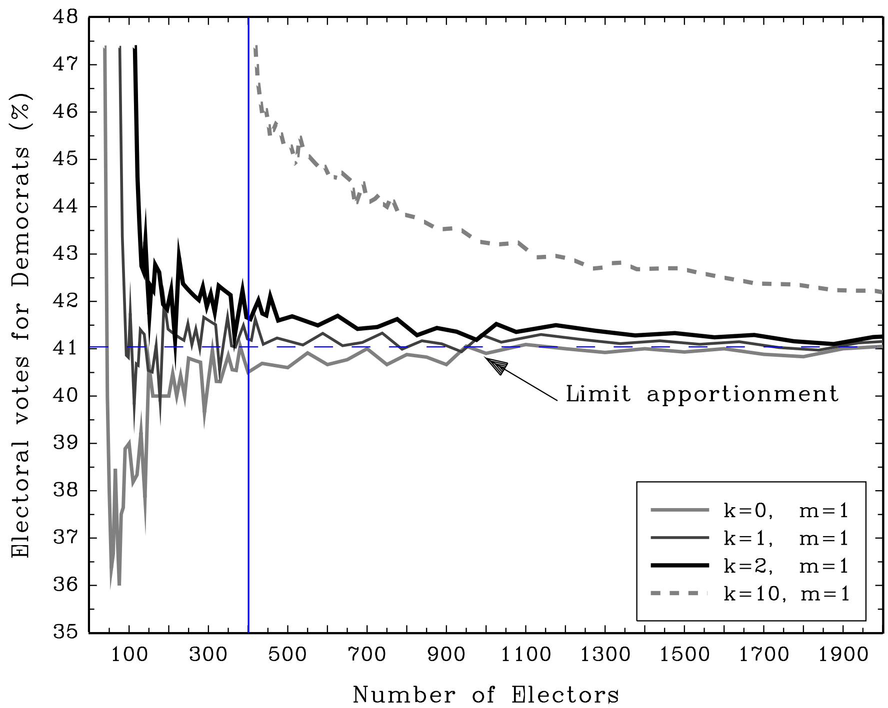

Figure 3: 1888 election.

Hillary Clinton's defeat in the Electoral College is "entrenched" and would have occurred under any of the variations that we consider. The only historical parallel is the election of 1888 which is Figure 4 in Barthélémy, Martin and Piggins (2014), and reproduced here as Figure 3. In 1888 Harrison received 233 electoral votes to Cleveland's 168, and so the vertical line in Figure 3 is drawn for a total of 401 electors. The limit apportionment in Figure 3 is 41.04%. Like Clinton, a larger House would not have been to Cleveland's advantage.

An immediate difference between Figures 2 and 3 is that the four representation graphs plotted in Figure 2 appear to be "inverses" of the corresponding graphs plotted in Figure 3. For example, take the $k = 10, m = 1$ graph. In Figure 2, this graph starts off beneath the limit apportionment electoral percentage and approaches it (from below) as House size increases. In Figure 3, the corresponding graph starts off above the limit apportionment electoral percentage and approaches it (from above) as House size increases. The fact that a representation graph approaches the limit apportionment electoral vote percentage as House size increases is a mathematical inevitability which we will explain in a moment. First, the difference between these two $k = 10$ graphs can be explained thus.[18] Trump won 30 states to Clinton's 20, so if we increase the number of electoral votes a state has by virtue of its Senators from 2 to 10, then Trump's "Senate" electoral votes increase from 60 to 300 and Clinton's increase from 40 to 200. The smallest size the Electoral College can be at this point is 561, with each state receiving one "House" electoral vote with eleven electoral votes allocated to the District of Columbia. This situation is no different to a federal situation in which a majority of states (including DC) is sufficient to determine the electoral winner without any weight given to their relative populations. In this scenario Clinton obtains $^{231}/_{561} \simeq 41.18\%$ of the electoral vote, which is why the graph starts off below the limit apportionment electoral vote percentage of 43.7%.

As House size increases, the importance of these "Senate" electoral votes declines relative to "House" electoral votes, and so the graph heads towards 43.7%. In 1888, Harrison carried 20 states to Cleveland's 18 and the states he carried accounted for a majority of the total apportionment population (approx. 58.96% to Cleveland's 41.04%).[19] A similar calculation to the one above shows that in the $k = 10, m = 1$ situation Cleveland obtains $^{198}/_{418} \simeq 47.37\%$ of the electoral vote, which is why the graph starts off above the limit electoral vote percentage of 41.04%. As before, the graph approaches 41.04% as House size increases. The explanation for the inverses,

[18] A similar explanation accounts for the "inverse" nature of the $k = 0, k = 1$ and $k = 2$ graphs in Figures 2 and 3.

[19] The states Trump carried account for approximately 56.3% of the apportionment population. Trump's situation and Harrison's are, therefore, similar in that both won a majority of states, and the states they won contained a majority of the total apportionment population.

then, is that Clinton's loss under federalism was *more* severe than her loss under a weighted voting system in which each state casts a number of votes equal to its population. For Cleveland, this loss was *less* severe (recall, he carried just under one-half of the states).

3 House size effect

When is there a House size effect? In an important paper, Miller (2014) identifies the following sufficient condition.[20]

Proposition. *Given any method of apportioning seats in the House into whole numbers, a Presidential election is subject to the House size effect if:*

(1) one candidate (say A) carries a majority of states, and

(2) the other candidate (say B) carries states that collectively hold a majority of the total apportionment population.

As a sufficient condition, a House size effect must arise as a matter of logic if conditions (1) and (2) are jointly satisfied. This sufficient condition was satisfied in both the 1876 and 2000 elections, but not in 1888 and 2016. In 2000 Bush carried 30 states to Gore's 21 (treating DC as a state) and so condition (1) is satisfied. As noted above, Gore's states accounted for approx. 51.68% of the total apportionment population and so condition (2) is satisfied. In 1876 Rutherford Hayes carried 21 states to Tilden's 17. However, Tilden's states accounted for 51.8% of the total apportionment population.[21] Again, conditions (1) and (2) are satisfied.

Therefore, Miller's sufficient condition shows that a House size effect must be present in both 1876 and 2000. Indeed, we have verified through our simulations the existence of a House size effect for the 2000 election in Figure

[20]This is Proposition 2 in Miller (2014) stated in its entirety. As mentioned in footnote 6, we have omitted any discussion of methods of apportionment in this note as it has no bearing on our observation about the 2016 election.

[21]Data from Table 1 in Miller (2014).

1, and the 1876 election is dealt with in Figure 5(a) in Barthélémy, Martin and Piggins (2014).[22]

The intuition behind the Proposition was alluded to in Section 2. When we are at the constitutional floor of one House seat per state, then the candidate who carries the majority of states (including DC) wins the electoral vote as each state casts three electoral votes (one "House" vote and two "Senate" votes). For example, in the 2000 election, this candidate was Bush. However, as House size increases, the importance of these two "Senate" electoral votes declines relative to the "House" electoral votes. As House size increases, further and further, we head towards the limit apportionment electoral vote percentage. At this point, the House has grown so large that, in effect, only "House" electoral votes count, and the electoral vote percentage (and outcome) is determined by the fraction of the apportionment population in the states carried by the candidates. As noted above, in the 2000 election, the candidate carrying states with a majority of the apportionment population was Gore. So, at some point in this process of making the House larger, the representation graph must crossover the 50% line and Gore is elected President.

Importantly, while (1) and (2) are sufficient conditions for a House size effect, they are not necessary conditions. The reason for this is that House seats must be allocated as whole numbers, not fractional numbers. If House seats could be allocated fractionally to states, then each state would receive its "quota" of seats, which is the perfect proportion of the 435 seats in the House of Representatives it is due owing to its share of the apportionment population. In such circumstances, Miller proves that (1) and (2) are necessary and sufficient for a House size effect.[23] This would make our simulation in Figure 2 unnecessary, as the failure of condition (2) to be satisfied would be

[22] The House size effect being present for the 2000 election is, of course, Neubauer and Zeitlin's original finding.

[23] This is Proposition 1 in Miller (2014). Note that fractional seats would eliminate the non-monotonicity observed by Neubauer and Zeitlin.

enough to know that there is no effect present for the 2016 election. However, given that seats are awarded in whole numbers, there is no way of knowing whether a House size effect is present or not in the 2016 election without running a simulation.

Figure 2 shows that Trump's election in 2016 was like Harrison's in 1888, and unlike Tilden's in 1876 and Bush's in 2000. The referendum paradox was entrenched. It is tempting to think that one way to avoid the referendum paradox is to abandon "winner-takes-all" and allow states to allocate their electoral votes fractionally, in proportion to the popular vote percentages in each state.[24] However, this does not in general overcome the paradox. For example, in the 2000 election, such a procedure would generate approximately 259.17 electoral votes for Bush compared with 258.27 for Gore.[25] As this example demonstrates, the presence of third-party candidates means that a majority winner in the Electoral College is unlikely under a fractional system.[26] For the 2016 election, fractional voting would have produced approximately 255.43 electoral votes for Clinton compared with 249.07 for Trump. So, unlike the election in 2000, Clinton (the popular vote winner) would emerge as the plurality winner in this case.

References

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[2] Barthélémy, F., Martin, M., Piggins, A., 2014. The architecture of the Electoral College, the House size effect, and the referendum paradox. Electoral Studies 34, 111-118.

[24] For example, if Clinton obtains 34.36% of the popular vote in Alabama she receives 3.0924 of Alabama's 9 electoral votes.

[25] Authors' calculations.

[26] Miller (2009) considers fractional voting.

[3] Miller, N. R., 2009. A priori voting power and the U.S. Electoral College. Homo Oeconomicus 26, 341-380.

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[7] Nurmi, H., 1998. Voting paradoxes and referenda. Social Choice and Welfare 15, 333-350.